UNCLASSIFIED

AD NUMBER
AD492601
CLASSIFICATION CHANGES
TO: **UNCLASSIFIED**
FROM: **RESTRICTED**
LIMITATION CHANGES
TO: **Approved for public release; distribution is unlimited. Document partially illegible.**
FROM: **Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; 02 MAY 1946. Other requests shall be referred to Ballistic Research Laboratory, Averdeen Proving Ground, MD. Document partially illegible.**
AUTHORITY
E.O. 10501 dtd 5 Nov 1953; BRL ltr dtd 22 Apr 1981

THIS PAGE IS UNCLASSIFIED

595

ABERDEEN PROVING GROUND
MARYLAND

ORDNANCE DEPARTMENT U.S.A.

BALLISTIC RESEARCH LABORATORY

REPORT

TAYLOR AND MACCOLL COMPUTATIONS

BY

RAYMOND A. TURETSKY

REPORT NO. 595

2 MAY 1946

ORDNANCE RESEARCH & DEVELOPMENT CENTER

PROJECT NO.__________

RESTRICTED

BALLISTIC RESEARCH LABORATORIES

REPORT NO. 595

Turetsky/emn
Aberdeen Proving Ground, Md.
2 May 1946

TAYLOR AND MACCOLL COMPUTATIONS

Abstract

In the present report, a method is illustrated of obtaining more precise values of shock wave angle, pressure, Mach number, and drag coefficient from a given cone angle and velocity past the cone. From the Rankine-Hugoniot shock wave equations a single equation is obtained which fixes the position of the shock wave.

Another term is obtained in Maccoll's Taylor expansion about the surface of the cone which permits the use of a larger interval of integration with the same degree of smoothness and accuracy previously obtained for smaller intervals.

RESTRICTED

By the assumption of axially symmetric, irrotational flow of a perfect compressible fluid, Taylor and Maccoll[1] obtained the following differential equations for the tangential and normal components of velocity, u and v respectively, for conical flow:

$$\frac{1}{c}\frac{d^2u}{d\theta^2}\left\{\frac{v^2}{c^2} - \frac{\gamma-1}{2}\left(1 - \frac{u^2}{c^2} - \frac{v^2}{c^2}\right)\right\} = \frac{\gamma-1}{2}\left\{1 - \frac{u^2}{c^2} - \frac{v^2}{c^2}\right\}\left\{\frac{v}{c}\cot\theta + \frac{2u}{c}\right\} - \frac{u}{c}\frac{v^2}{c^2}$$

$v = du/d\theta$, where θ is the azimuth angle.

Only numerical integration has thus far proved feasible in solving these equations. It then is convenient to integrate out from the surface of the cone by assuming a certain value of u/c along it. In order to perform the first few steps of the integration, Macçoll[2] used the following series without the term in $\overline{\delta\theta}^6$, developed along the surface of the cone:

$$(u/c)_\theta = (u/c)_s\left[1 - \overline{\delta\theta}^2 + a\frac{\overline{\delta\theta}^3}{3} - b\frac{\overline{\delta\theta}^4}{4} + c\frac{\overline{\delta\theta}^5}{5} - d\frac{\overline{\delta\theta}^6}{6}\right]$$

$$(v/c)_\theta = -2(u/c)_s\,\overline{\delta\theta}\left[1 - a\overline{\delta\theta} + b\frac{\overline{\delta\theta}^2}{2} - c\frac{\overline{\delta\theta}^3}{2} + d\frac{\overline{\delta\theta}^4}{2}\right],$$

where the subscript s denotes a value taken at the surface of the cone and, letting $(u/c)_s = U_s$.

$$a = \cot\theta_s$$

$$b = \cot^2\theta_s + 8U_s^2/3(\gamma-1)(1-U_s^2)$$

$$c = a(7/12 + \cot^2\theta_s + 20U_s^2/3(\gamma-1)(1-U_s^2)).$$

$$d = \cot^4\theta_s + .95\cot^2\theta_s + .133333 - \frac{4U_s^2}{1-U_s^2}\left[5.02058\cot^2\theta_s + 6.00925\frac{4U_s^2}{1-U_s^2} + .823045\right]$$

[1] Taylor and Maccoll - Proc. Roy. Soc. A 139 278
[2] Maccoll - Proc. Roy. Soc. A 159 459

Numerical integration can then be used until θ_w, the shock wave angle is reached* But this is precisely where difficulties arose in the past. The conditions determining the existence of a shock wave are obtained from the Rankine-Hugoniot equations. In the form obtained by Taylor and Maccoll[3], these involved the use of three sets of graphs, making the computation not only cumbersome, but inaccurate as well. R. N. Thomas[4] used an improved version involving the use of only one graph. In this report an even simpler condition will be derived, which obviates the use of graphs entirely.

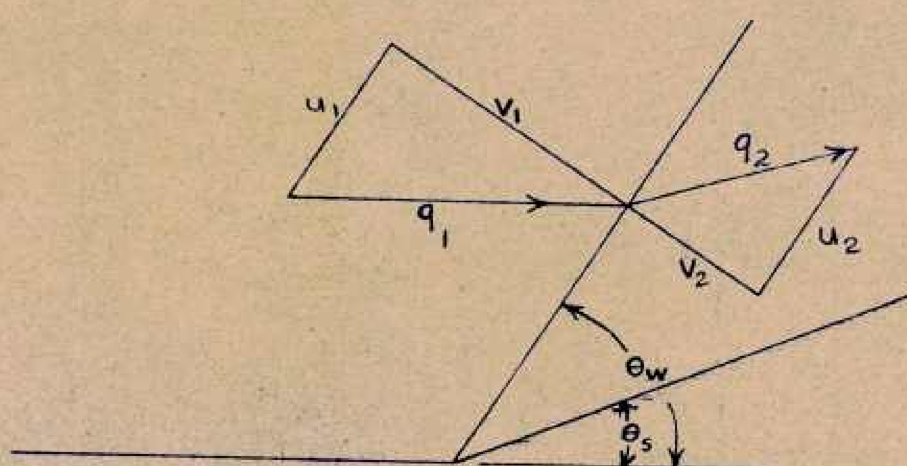
From the attached diagram and the Rankine-Hugoniot equations,

$$u_1 = u_2$$

$$v_2\rho_2 = v_1\rho_1$$

$$p_2 - p_1 = \rho_1 v_1 (v_1 - v_2)$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma-1)p_1 + (\gamma+1)p_2}{(\gamma-1)p_2 + (\gamma+1)p_1}$$



* This implies that the free-stream velocity vector is parallel to the axis of the cone.

[3] Taylor and Maccoll - ibd.

[4] Richard N. Thomas - BRL Report No. 483.

we can, using the transformations

$$\frac{v_2}{a_1} = \tau \ , \ \frac{v_1}{a_1} = \sigma \ , \text{ where } a_1^2 = \gamma \frac{p_1}{\rho_1} \ , \quad a_1$$

being the velocity of sound in the undisturbed air, obtain the equations

$$\frac{2}{\sigma} = (\gamma+1)\,\tau - (\gamma-1)\,\sigma \tag{1}$$

$$\frac{\gamma+1}{2}\,\sigma - \frac{\gamma-1}{2}\,\tau = \frac{p_2}{\sigma p_1} \tag{2}$$

Introduce the quantity

$a_2^2 = \gamma \dfrac{p_2}{\rho_2} = \dfrac{\gamma-1}{2}(c^2-q_2^2)$, a_2 being the velocity of sound behind the shockwave.

Then $\dfrac{a_2^2}{a_1^2} = \dfrac{p_2}{p_1}\,\dfrac{\tau}{\sigma}$

and

$$\frac{p_2}{\sigma p_1} = \frac{\gamma-1}{2}\,\frac{c^2-q_2^2}{v_2^2}\,\tau \ .$$

Substituting in eq. (2) gives

$$\frac{\tau}{\sigma} = \frac{\gamma+1}{\gamma-1}\,\frac{v_2^2}{c^2-u_2^2} \tag{3}$$

Solving equations (1), (2), and (3), for $\frac{1}{\sigma^2}$ and $\frac{p_1}{p_2}$ gives

$$\frac{1}{\sigma^2} = \frac{(\gamma+1)^2}{2(\gamma-1)}\,\frac{V_2^2}{1-U_2^2} - \frac{\gamma-1}{2} \tag{4}$$

$$\frac{p_1}{p_2} = \frac{4\gamma}{\gamma^2-1}\,\frac{V_2^2}{1-U_2^2-V_2^2} - \frac{\gamma-1}{\gamma+1} \tag{5}$$

where $U_2 = u_2/c$, $V_2 = v_2/c$.

But, from the diagram and eq. (3)

$$\cot\theta_w = -\frac{u_1}{v_1} = -\frac{u_2}{v_2}\,\frac{\tau}{\sigma} = -\frac{\gamma+1}{\gamma-1}\,\frac{u_2 v_2}{c^2-u_2^2} \tag{6}*$$

Eq. (6) suffices to determine when the integration must stop, for it can be written in the form

$\cot\theta_w + \frac{\gamma+1}{\gamma-1}\,\frac{U_2\,V_2}{1-U_2^2} = 0$, showing that inverse interpolation may be used to determine the value of θ_w. Once θ_w is determined, U_2 and V_2 may be found by direct interpolation.

It remains to find the Mach number M and the head drag coefficient K_D. Obviously, $M = q_1/a_1 = v_1 \csc\theta_w/a_1 = \sigma\csc\theta_w$ and can thus be found by using eq. (4).

Also,

$$K_D = \frac{\frac{\pi}{4}\,d^2(p_s-p_1)}{\rho d^2 q_1^2}$$

$$= \frac{\pi}{4}\,\frac{1}{\gamma M^2}\left\{\frac{p_s}{p_1} - 1\right\} \tag{7}$$

But

$$\frac{p_s}{p_1} = \frac{p_s}{p_2}\,\frac{p_2}{p_1}.$$

Since the flow is adiabatic behind the shock wave,

$\frac{p_s}{p_2} = \left(\frac{\rho_s}{\rho_2}\right)^{\gamma}$ while

* After this manuscript had been completed, it was found that this equation was used by the Navy in their calculations.

$$\frac{c^2-q_s^2}{c^2-q_2^2} = \frac{p_s}{\rho_s} \div \frac{p_2}{\rho_2} = \frac{p_s}{p_2} \; \frac{\rho_s}{\rho_2} = \left(\frac{p_s}{p_2}\right)^{1-\frac{1}{\gamma}}$$

Thus $$\frac{p_s}{p_2} = \left(\frac{c^2-q_s^2}{c^2-q_2^2}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{1-U_s}{1-U_2^2-V_2^2}\right)^{\frac{\gamma}{\gamma-1}}$$

and $$\frac{p_s}{p_1} = \left(\frac{1-U_s^2}{1-U_2^2-V_2^2}\right)^{\frac{\gamma}{\gamma-1}} \div \frac{p_1}{p_2}, \quad \text{where} \quad \frac{p_1}{p_2}$$

can be determined from eq. (5). Finally K_D is found by substituting in (7).

It should be observed that u_s/c is not known in practice while M is. This difficulty is surmounted by solving the differential equations for u/c and v/c for a variety of values of u_s/c and the same θ_s. In this way sets of values for θ_w, M, and K_D are obtained and θ_w and K_D can be plotted against M. Thus the shock wave angle and head drag coefficient can be obtained if the velocity and cone angle of the projectile are known.

Such curves were actually obtained for the four values of θ_s : 9.5°, 12.1°, 15°, and 20° and are shown at the end of the report. It was found convenient to use the following values of u_s/c in each case: .35 , .36, .375, .4, .45, .5, .6, .7, .8, .9. It will be seen that in the case θ_s = 9.5°, there are two values of K_D and θ_w for some values of M. If lower values of u_s/c were used, complete sets of double values could be obtained. However, in practice the upper part of the curve does not usually occur, so it can be safely eliminated.

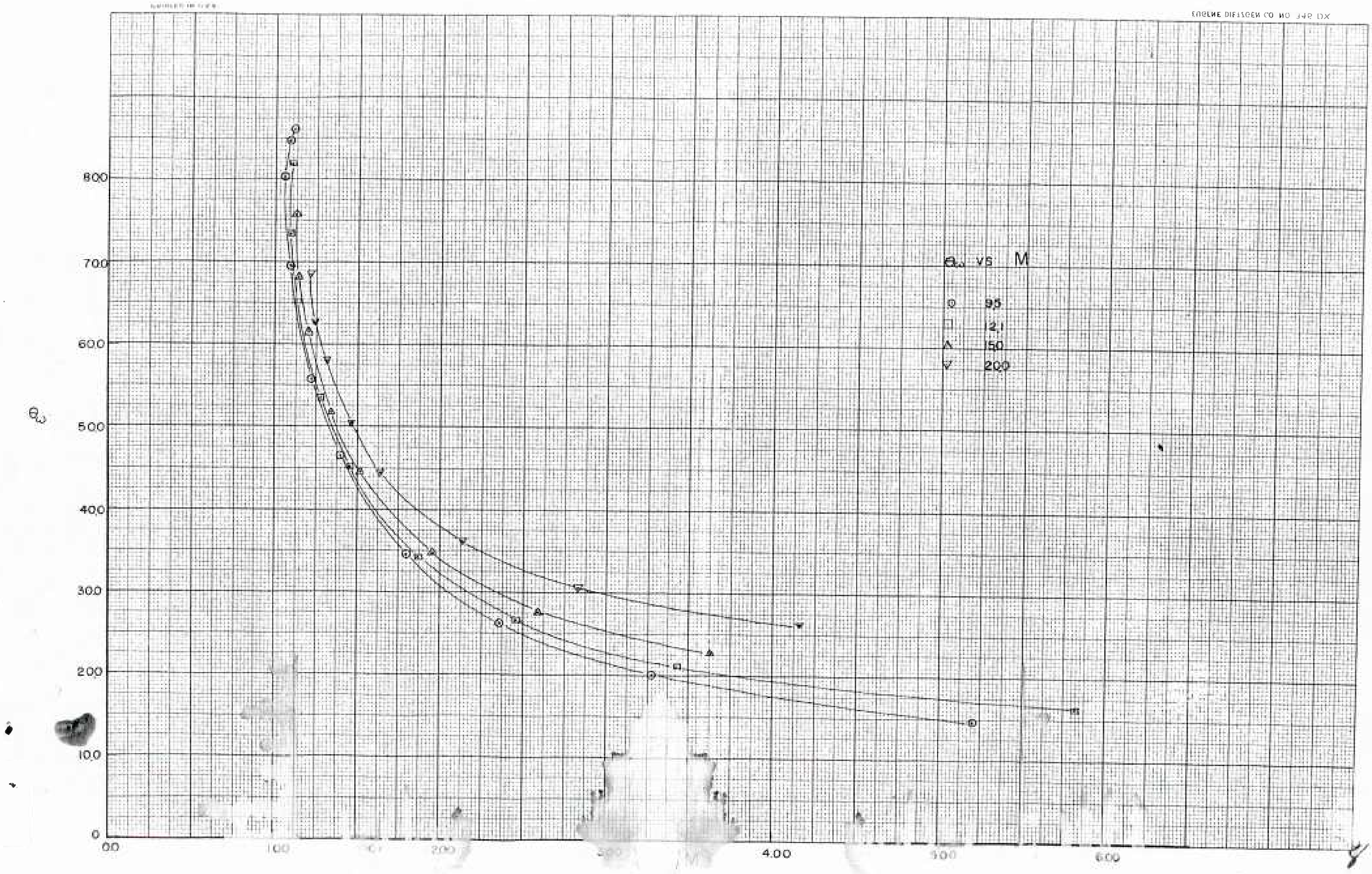
It may also be mentioned that the integration was carried out in the manner used for a trajectory rather than in the equal interval method originally used by Taylor and Maccoll. The former has the advantage of permitting a small interval to be used at the beginning, which can be

later increased as much as the problem permits. Since the series for u/c and v/c give correct values to within $.0_65$ for $\theta \geq 9.5°$, $|\delta\theta| \leq 1°$, it is advisable to begin the integration with an interval of 0.5°. We thus obtain values of u/c and v/c for $\theta = \theta_s-1.0$, $\theta_s-0.5$, θ_s, $\theta_s+0.5$, $\theta_s+1.0$. This gives five lines with fourth differences, sufficient for carrying out the integration.

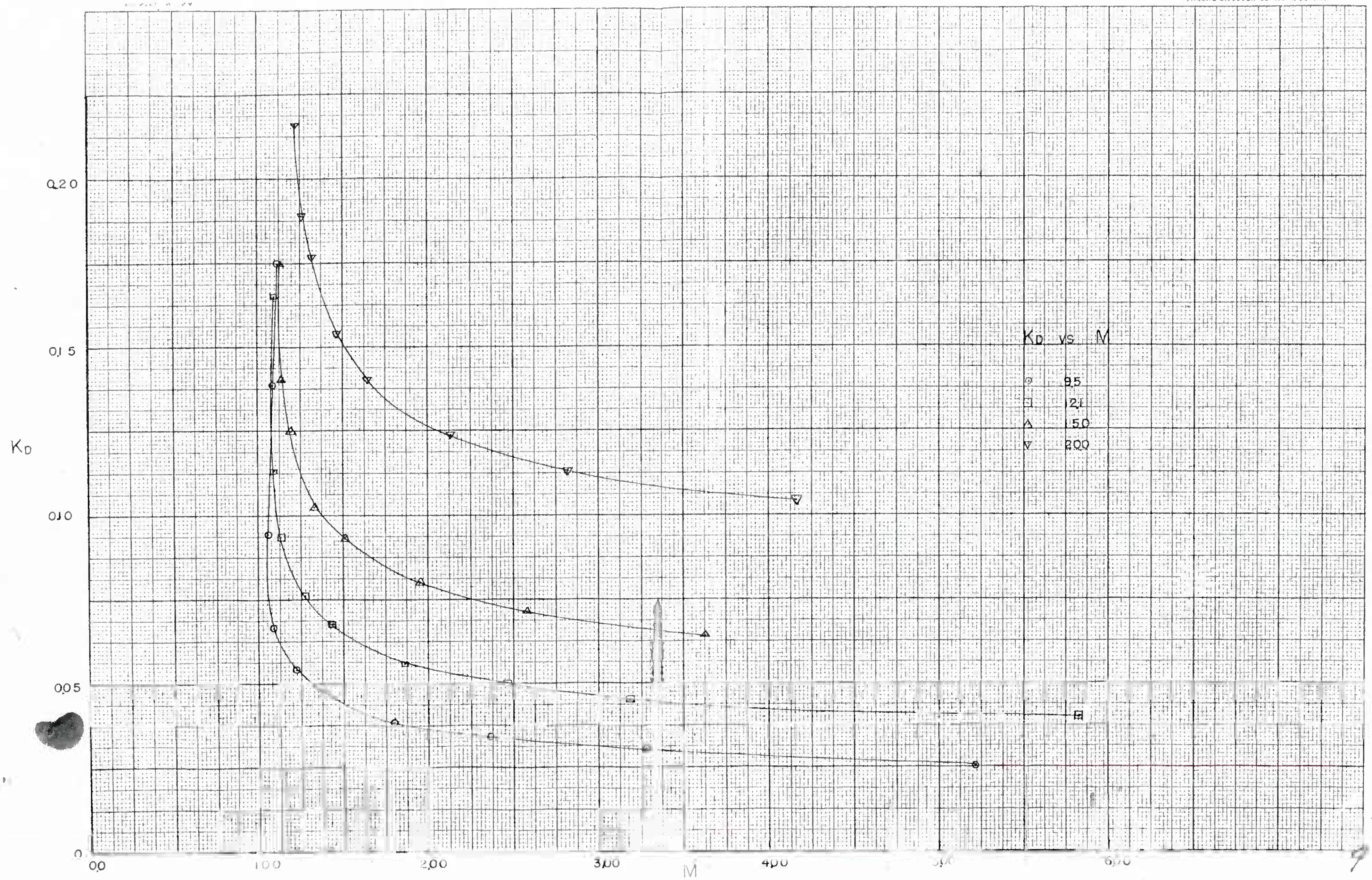
In general, it was possible to extend the interval of integration up to 4°, greater than any used by Taylor and Maccoll, which naturally shortened the computation. In some cases, notably for $\theta_s = 9.5°$, $u_s/c = .375$, .4; and $\theta_s=12.1°$, $u_s/c = .375$, the interval of integration had to be drastically reduced just before the integration was completed, since these correspond to Mach numbers close to 1. For the 9.5° cone, the Mach number can be 1.04 and still yield a solution; i.e., permit the shock wave to remain attached to the cone (see graphs). This, however, is a theoretical result which could not be expected to hold in practice, since the assumption of irrotational flow is not valid in the transsonic region.

Raymond A. Turetsky

Raymond A. Turetsky



θ_ω VS M
95
12.1
150
200
θ_ω
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0
0.0
1.00
2.00
3.00
4.00
5.00
6.00
M



0.20
0.15
0.10
0.05
0
KD
0.0
1.00
2.00
3.00
4.00
6.00
M
KD vs M
9.5
12.1
15.0
20.0